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Heat Transfer Coefficients for a Hot Gas Oscillating at High Amplitudes in a Cylindrical Chamber

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It has been noted that heat transfer from a fluid to a tube is much enhanced if the fluid is vibrating (*1, 7, 9*). This enhancement is of interest for several reasons, one of which is the use of oscillations to increase the efficiency of a heat exchanger. Also flame-generated oscillations can occur in ramjet engines, and the increased heat transfer may be detrimental to the durability of the combustion chamber.

Furthermore a phenomenon known as *oscillatory combustion* is encountered in the course of many rocket development programs. This phenomenon occurs when the combustion process supports oscillations of the combustion gas in the rocket motor. Usually the oscillations are in acoustic modes of the combustion chamber and may be sensed by high-frequency response pressure-recording instrumentation. Frequently these oscillations are driven to very high amplitudes, and they are harmful to both the performance of the motor and its structural integrity. Part of the harm results from increased heat transfer, part from the change in mean pressure caused by interaction of the oscillations and combustion process, and part from the hammering action of the oscillations.

While there have been measurements made of the heat transfer coefficient which exists in a combustion tube in the presence of low-amplitude oscillations (*9*), such measurements taken in the presence of high amplitude oscillations are lacking. This is because high-amplitude oscillations are generally encountered in the combustion system of rocket motors, and in the presence of the high temperature, chemically-reactive environment which exists in the rocket combustion chamber measurements of all kinds are difficult to make. Measurements of the heat transfer coefficient would be desirable because they would aid in hardware design, and they would also augment the present heat transfer literature.

This paper presents some estimates of the average heat transfer coefficient which exists in the presence of a high-temperature gas oscillating at high amplitudes in a cylindrical steel cavity. The measurements were taken over the frequency range of 600 to 8,000 cycles/sec. and in the presence of decaying oscillations which ranged approximately from a peak to peak amplitude of 30 down to 2 lb./sq.in.

EXPERIMENT

Inasmuch as the treatment of the data is so dependent upon the special conditions which exist in this experiment, the experiment will be described before the data reduction is described.

The combustor used in the experiments (see Figure 1 and reference 5) is a heavy walled, side vented, cylindrical steel chamber which is connected to a pressure surge tank by means of a short pipe that contains a check valve. A disk of solid propellant is placed in each end of the chamber, and following ignition the combustion of the propellant produces rather severe first mode axial oscillations which persist throughout the firing and for some time after the propellant is consumed. The frequency of the first-mode oscillations is determined by the length of the combustion chamber which may be varied from test to test.

The diameter of the pipe which connects the combustor and the surge tank is large enough that during the firing the gas flow in the pipe is well below sonic velocity. Therefore while the propellant is burning, the mean pressure in the combustion chamber is approximately the same as that in the surge tank, which was 200 lb./sq. in. gauge in this series of tests. After the propellant is consumed, the check valve closes and the mean pressure in the combustion chamber decreases at a rate fixed by the heat transfer from the hot combustion gas to the steel walls.

The above type of test can be used to obtain the heat transfer under oscillating conditions, but it is desirable that heat transfer be measured also when the gas is not oscillating. For this purpose the burner can be mechanically stabilized by the use of a baffle (2) and this type of firing used to obtain non-oscillating heat transfer information.

DATA REDUCTION

Because of the difficulties inherent in any technique used to measure the gas and wall temperatures it was decided that a technique be used which did not require the measurement of these temperatures but rather was dependent upon the determination of the mean chamber pressure. Only the period of time immediately following the burnout of the propellant is considered. The gas in the burner during that interval has no mean flow field and may or may not be oscillating, depending on whether or not the burner contains a baffle.

It may be assumed that heat transfer from the gas to the chamber walls can be described by the equation

$$\frac{dQ}{dt} = hA(T - T_w) \qquad (1)$$

This equation contains the assumption that there is an effective average wall temperature. The wall temperature is by no means uniform, since the propellant disks insulate the end walls until burnout while the cylindrical wall is heated throughout the course of the burn. Furthermore it seems reasonable that the heat transfer to a point on the cylindrical wall (and therefore also the temperature of the point) would be dependent upon the location of the point with respect to the nodal planes of the oscillations.

There are other assumptions implicit in Equation (1). The first is that h is not strongly dependent upon the amplitude of the oscillations which are decaying during the period of time in which this analysis is concerned. As will be shown later the experimental results seem to confirm the validity of this assumption. The second implicit assumption is that the oscillating gas temperature can be represented by an average value T. This average also considers the variation along the length of the chamber.

A heat balance shows also that

$$\frac{dQ}{dt} = -m_g C_v \frac{dT}{dt} \qquad (2)$$

and combination of Equations (1) and (2) gives

$$\frac{dT}{dt} = \frac{-hA}{m_g C_v}(T - T_w) \qquad (3)$$

If any heat generated or absorbed by chemical reaction is neglected, one may use the perfect gas law to obtain the expression

$$\frac{d\left(\frac{PV}{nR}\right)}{dt} = -\frac{hA}{m_g C_v}\left(\frac{PV}{nR} - T_w\right) \qquad (4)$$

or

$$\frac{dP}{dt} = -\frac{hA}{m_g C_v}\left(P - T_w \frac{nR}{V}\right) \qquad (5)$$

In general the wall temperature as a function of time can be described by the series

$$T_w = T_0 + T_1 t + T_2 t^2 + \ldots + T_j t^j$$

and substitution of this equation into Equation (5) yields the differential equation

$$\frac{dP}{dt} = -\frac{hA}{m_g C_v}\left[P - \frac{nR}{V}(T_0 + T_1 t + T_2 t^2 + \ldots + T_j t^j)\right] \qquad (6)$$

The solution of this equation is

$$P - \frac{nR}{V}\sum_{m=o}^{m=j}\frac{t^{j-m}}{(j-m)!}\left\{\sum_{\epsilon=o}^{\epsilon=m}(-1)^{m+\epsilon}(j-\epsilon)!\left(\frac{m_g C_v}{hA}\right)^{m-\epsilon} T_{j-\epsilon}\right\} = \left(P_i - \frac{nR}{V}K\right)e^{\frac{-hA}{m_g C_v}t} \qquad (7)$$

where

$$K = \sum_{\epsilon=o}^{\epsilon=j}(-1)^{j+\epsilon}(j-\epsilon)!\left(\frac{m_g C_v}{hA}\right)^{j-\epsilon} T_{j-\epsilon}$$

and P_i is the chamber pressure when t is equal to zero, the time when the burning ceases. If the wall temperature as a function of time can be described by a quadratic equation, Equation (7) reduces to

$$P - \frac{nR}{V}\left\{2\left(\frac{m_g C_v}{hA}\right)^2 T_2 - \frac{m_g C_v}{hA}T_1 + T_0 + \left(T_1 - 2\frac{m_g C_v}{hA}T_2\right)t + T_2 t^2\right\} = \left(P_i - \frac{nR}{V}K'\right)e^{\frac{-hA}{m_g C_v}t} \qquad (8)$$

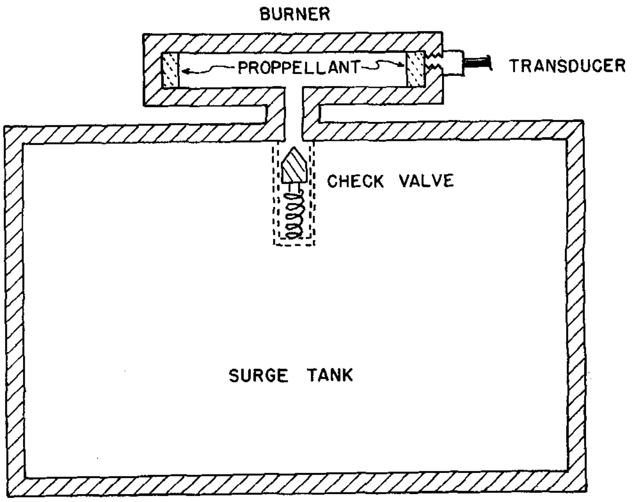

Fig. 1. Schematic illustration of combustion chamber and instrumentation.

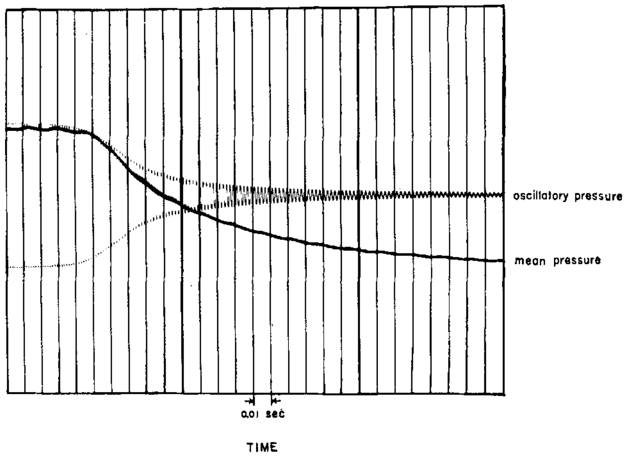

Fig. 2. Typical test record.

where

$$K' = 2\left(\frac{m_gC_v}{hA}\right)^2 T_2 - \frac{m_gC_v}{hA} T_1 + T_0$$

If one optimistically assumes that the wall temperature is constant over the tenth of a second period of concern, Equation (7) can be written as

$$\ln\left(P - \frac{nR}{V} T_0\right) - \ln\left(P_i - \frac{nR}{V} T_0\right) = -\frac{hA}{m_gC_v} t \quad (9)$$

It is now worthwhile to see if this equation will describe the experimental data. If the wall temperature were constant, a plot of $\ln\left(P - \frac{nR}{V} T_0\right)$ against time would be a straight line with a slope of $-\frac{hA}{m_gC_v}$. To test this possibility the pressure is measured at 0.01-sec. intervals from a typical test record such as that shown in Figure 2. A value for $\frac{nR}{V} T_0$ is estimated and a plot made of $\ln\left(P - \frac{nR}{V} T_0\right)$ vs. t. If the resultant plot is not a straight line, a new value for $\frac{nR}{V} T_0$ is estimated and a new plot made. As Figure 3 shows there exists a value for $\frac{nR}{V} T_0$ which yields a straight line.

One may not yet conclude that the wall temperature is indeed constant and that the value of $\frac{nR}{V} T_0$ which yields the straight line is correct. First one must determine whether or not the straight line can also be a product of varying wall temperature as described by Equation (8). Observation of this equation [or Equation (7)] shows that a plot of $\ln\left(P - \frac{nR}{V} T_0\right)$ vs. t yields a straight line only if T_1, T_2, etc. are small and the wall temperature is nearly constant. Therefore the existence of a straight line supports the constant wall temperature assumption.

It is conceivable that h, C_v, and T_w could vary simultaneously in such a manner that a straight line would be

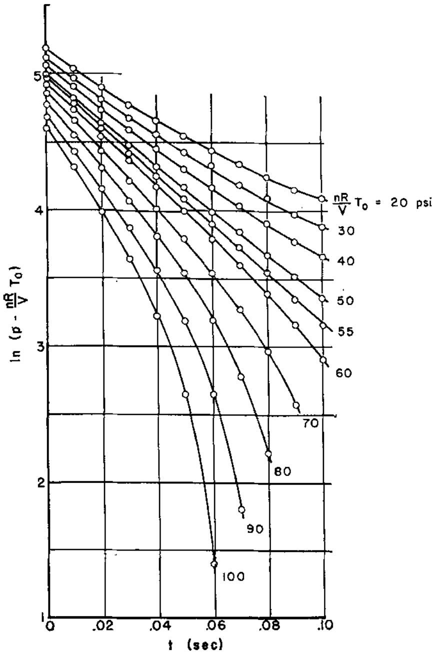

Fig. 3. This figure illustrates the use of the experimental data with Equation (9). As can be seen the straightness of the line (but not its slope) is a rather sensitive function of the assumed value of $nR/V\ T_0$.

obtained although the value of $\frac{nR}{V} T_0$ was not meaningful. While this simultaneity seems improbable, its possibility cannot be neglected. Therefore an estimate is made of the variation in T_w which might occur during the period of interest. For this purpose consider a ½-in. slab of steel containing a linear temperature gradient at time zero. If the hot side of the slab is insulated and the cold side is maintained at a fixed temperature, the high-temperature side accomplishes 10% of its ultimate temperature drop in about 0.125 sec. This slab heat transfer is rather analogous to that which occurs in the wall of the combustor used in this experiment. Since a 10% variation in T_w would cause variations in h which would be within the scatter of the data, it is felt that the assumed constant value of the average wall temperature is a good approximation.

Therefore for each experimental firing the $\ln\left(P - \frac{nR}{V} T_0\right)$ is plotted as a function of time for several assumed values of $\frac{nR}{V} T_0$. The value which yields the best straight line is taken as correct and the value of the slope $\left(-\frac{hA}{m_gC_v}\right)$ measured. The area (A) is known from the chamber geometry, C_v is evaluated from an assumed value of $\gamma(1.22)$, and m_g is calculated as follows: it is known that $c = \left\{\frac{\gamma PV}{nM}\right\}^{1/2}$, but $m_g = nM$ and so $m_g = \frac{\gamma PV}{c^2}$.

The value of c (94,000 cm./sec.) is obtained from the measured frequency (f), the chamber length, and the relation $2Lf = c$. Thus the value of h is calculated from the

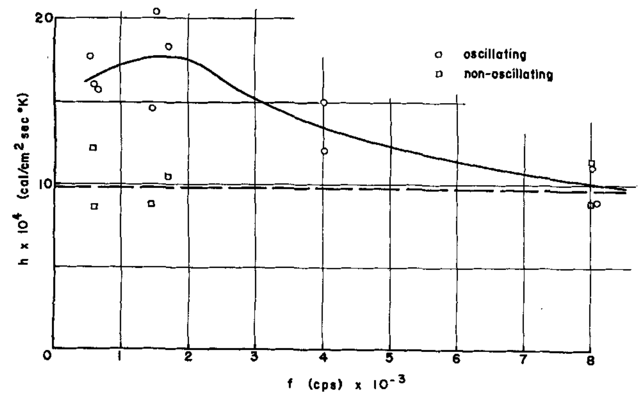

Fig. 4. The values of the heat transfer coefficients as a function of the first axial mode frequency of the burner cavity which is one-half wave length long.

measured slope, an accurate estimate of γ, the chamber geometry, and the measured speed of sound.

RESULTS AND DISCUSSION

As one might expect, the heat transfer coefficient for the nonoscillating tests was not a function of burner length. Figure 4 shows this lack of dependency. The average value for the nonoscillating heat transfer coefficient was 9.9×10^{-4} cal./sq. cm. sec. °K.

Figure 4 shows also the average heat transfer coefficient for the oscillating tests. Most interesting is the rather strong frequency dependence, since some investigations (*9, 6, 4*) found little relation between the frequency and the heat transfer. The results of (*4*) must be regarded skeptically because of an incorrect analysis which is otherwise somewhat similar to the analysis presented in this paper. The trend does however agree with that observed by Tailby (*8*).

In the derivation of Equation (8) it was assumed that the value of h was relatively independent of the amplitude of the oscillations. This assumption is supported by the results of Harrje (*3*) who found that the oscillations he observed (as great as 100 lb./sq.in. peak to peak) increased the heat transfer by a maximum of 10%. Jackson et al. (*6*) also observed a rather weak dependence of heat transfer upon the sound level. Zartman and Churchill (*9*) however observed that the heat transfer coefficient increased linearly with the amplitude of the oscillations, and they observed a maximum increase of about 100%. If the dependence they observe were extrapolated to the amplitudes encountered in this study, the oscillations should increase the value of h by a factor of about 26. Such obviously was not the case. They observe that oscillations of a very much smaller amplitude (about 0.7 lb./sq.in. peak to peak) double the heat transfer coefficient. Inasmuch as that is the maximum effect observed in this study also, the increase apparently reaches a saturation level with amplitude. These factors then seem to justify the assumption that h is constant over the variation of amplitude which occurs while a determination is made.

There are several approximations involved in the derivation of Equation (8) and its use in the simplified form of Equation (9). While some of these approximations are not too accurate, the results are obtained from the measurement of pressure alone. It is felt that the precision of this method which relies upon the measurement of pressure (which can be measured within 2%) is great enough to justify the use of this technique. The obvious alternate technique calls for measurements of heat transfer, combustion gas temperature, and wall temperature. The cumulative error inherent in determining the heat transfer coefficient from these measurements (which are difficult to make) would probably cause any results to be at least as uncertain as those obtained by the use of the much simpler method.

SUMMARY

It is found that the convective heat transfer coefficient for a gas oscillating at high amplitudes is similar to that observed at lower amplitudes. That is the oscillations increase the heat transfer coefficient with the increase being frequency dependent. While the coefficient is approximately doubled at 1,000 cycles/sec., higher frequencies are less effective in promoting heat transfer, and at about 10,000 cycles/sec. the oscillations completely lose their effect.

This study was concerned only with average heat transfer coefficients. The authors feel that a detailed study of oscillatory heat transfer can reveal a great deal about the interaction of oscillations and boundary layers. Such a study, although difficult, seems well worth undertaking because it would reveal much information which could be used to evaluate the extensive theoretical work which has been done in the field.

NOTATION

A = area of combustion chamber walls
c = speed of sound in the combustion gas
C_v = heat capacity of combustion gas at constant volume
f = frequency of oscillation
h = heat transfer coefficient
j, m, ϵ = indexes of summation
L = combustion chamber length
m_g = mass of gas in combustion chamber
M = molecular weight of combustion gas
n = number of moles of gas in combustion chamber
P = mean chamber pressure
P_i = mean chamber pressure at time zero
Q = heat
R = universal gas constant
t = time
T = average temperature of the gas in the burner during one cycle of oscillations
T_w = average temperature of combustion chamber wall inner surface
$T_0, T_1, T_2, \ldots T_j$ = coefficients describing variance of T_w with time
V = volume of combustion chamber
γ = specific heat ratio for the combustion gas

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